Relativistic Approach to Superfluidity in Nuclear Matter

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Abstract

Pairing correlations in symmetric nuclear matter are studied within a relativistic mean–field approximation based on a field theory of nucleons coupled to neutral (σ and ω) and to charged (ϱ) mesons. The Hartree–Fock and the pairing fields are calculated in a self–consistent way. The energy gap is the result of a strong cancellation between the scalar and vector components of the pairing field. We find that the pair amplitude vanishes beyond a certain value of momentum of the paired nucleons. This fact determines an effective cutoff in the gap equation. The value of this cutoff gives an energy gap in agreement with the estimates of non relativistic calculations.

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I. INTRODUCTION

In the investigation of superfluidity or superconductivity phenomena in systems of strongly interacting fermions a crucial role is played by the gap equation of the BCS theory [1]. For infinite homogeneous systems the kernel of the gap equation is usually a slowly decreasing function of the quasiparticle momentum. Even if the sum over momenta converges, contributions from terms involving large momentum components of the interaction and quasiparticle energies very far from the Fermi surface, are not negligible [2], despite the fact that the gap energy may result to be a small fraction of the Fermi energy. Contrary to what happens for electrons in metals, for nuclear or neutron matter, there is no natural cutoff over momentum. Consequently for nuclear matter a relativistic treatment is desireable. In the present work we investigate pairing correlations in cold symmetric nuclear matter by taking into account relativistic effects in a consistent way. A first study in this direction, for $^1S_0$ pairing in nuclear matter, has been already performed by H. Kucharek and P. Ring [3] within the framework of the Quantum Hadrodynamics theory of Ref. [4] (QHD). These authors, though introducing some simplifying assumptions (e.g exchange terms of the interaction have been neglected), have shown that pairing correlations and partcle–hole correlations can be described on the same foot within a relativistic treatment. In our opinion the consequences of a relativistic quantum approach have not been thoroughly exploited in Ref. [3]. For instance, the amplitudes of the nucleon field in the quasiparticle states have been assumed to be proportional simply to the four–spinors of the Hartree approximation.

In a recent paper [5] a more complete relativistic treatment of the Hartree–Fock–Bogoliubov approximation to QHD has been presented. The authors of Ref. [5] have shown that the pairing field has large scalar and vector components. The actual value of the gap in the excitation spectrum is the result of a strong mutual cancellation between these components of the pairing field. In the present work we obtain similar results, in particular the expression for the quasiparticle energy practically coincides with that of Ref. [5]. However, though the starting point of our treatment is the same as in Ref. [5], i.e. a relativistic gen-

eralization of the Gorkov scheme [6], we do not need to introduce any particular "ansatz" for an effective single–particle Lagrangian, as done in Ref. [5]. Moreover, we show that a more consistent treatment, from a relativistic point of view, brings out a new interesting feature: the pair amplitude vanishes beyond a certain momentum of the paired nucleons. The value of this momentum depends on the nuclear density and on the strength of the self–consistent pairing field. As a consequence in the relativistic gap equation a natural cutoff over momentum occurs. In the previous works of Refs. [3,5] it has been necessary to introduce such a cutoff in an arbitrary way, in order to obtain a satisfactory agreement with the current estimates of nonrelativistic calculations [2,9–14] for the energy gap.

We base our approach on the version I of QHD (QHD–I) [4], we include also the charged ϱ meson field in a phenomenological way similar to the MFT approach to QHD–II of Ref. [4]. For simplicity we neglect the pion field. Adding the pions would produce only small quantitative differences in our results [5].

Here we are mainly interested in studying new effects introduced by a relativistic approach to superfluidity. First we perform our calculations by using the same approximations already introduced in Refs. [7] and [8] for studying collective modes and response functions of nuclear matter. These approximations amount to neglecting finite–range and retardation effects in the exchange of mesons between nucleons. Within this approach we obtain simple expressions for the relevant quantities, where we can easily appreciate the role played by the various ingredients of the theory. In spite of the approximations introduced, the essential features of a relativistically covariant treatment are retained.

The value of the energy gap obtained with the approach just mentioned is much larger than the value predicted by nonrelativistic treatments [2,9–14]. A better quantitative agreement can be achieved by taking into account the finite range of the nucleon–nucleon interaction. Retardation effects instead do not play a significant role. We evaluate the finite–range effects by using an iterative procedure and show that the first iteration gives already a sufficiently accurate approximation.

II. FORMALISM

In the model adopted here the nucleons are coupled to neutral mesons (σ and ω) and to the charged vector meson, ϱ. According to the usual procedure employed in studying superconductivity we consider ensembles with indefinite number of particles. Therefore we add to the Lagrangian the term $\mu\ \bar{\psi}(x)\gamma^0\psi(x)$, where μ is the chemical potential ($\psi(x)$ denotes the 8–component nucleon field). In the end the value of the chemical potential will be determined by fixing the average baryon density. With these ingredients the field equations are [4]

$$(i\partial_\lambda\gamma^\lambda - g_V V_\lambda(x)\gamma^\lambda - g_\varrho \mathbf{B}_\lambda(x)\cdot\boldsymbol{\tau}\gamma^\lambda + \mu\gamma^0 - M + g_S\Phi(x))\psi(x) = 0\ , \tag{2.1a}$$

$$(\partial_\lambda\partial^\lambda + {m_S}^2)\Phi(x) = g_S\bar{\psi}(x)\psi(x)\ , \tag{2.1b}$$

$$\partial_\lambda W^{\lambda\nu}(x) + {m_V}^2 V^\nu(x) = g_V\bar{\psi}(x)\gamma^\nu\psi(x)\ , \tag{2.1c}$$

$$\partial_\lambda \mathbf{L}^{\lambda\nu}(x) + {m_\varrho}^2\mathbf{B}^\nu(x) = g_\varrho\bar{\psi}(x)\gamma^\nu\boldsymbol{\tau}\psi(x)\ , \tag{2.1d}$$

where $W^{\lambda\nu}(x) = \partial^\lambda V^\nu(x) - \partial^\nu V^\lambda(x)$ and $\mathbf{L}^{\lambda\nu}(x) = \partial^\lambda\mathbf{B}^\nu(x) - \partial^\nu\mathbf{B}^\lambda(x)$. The quantities $\Phi(x)$, $V^\nu(x)$ and $\mathbf{B}^\nu(x)$ represent the scalar, vector and charged vector fields, respectively. In the present paper we are concerned with a uniform system at equilibrium, it is convenient to study such a system through the one–particle density matrix in four–momentum space:

$$\widehat{F}_{\alpha\beta}(p) = \frac{1}{(2\pi)^4}\int d^4R\ e^{-ip\cdot R} < 0| : \bar{\psi}_\beta(\frac{R}{2})\psi_\alpha(-\frac{R}{2}) : |0>\ , \tag{2.2}$$

where α and β are double indices for spin and isospin. The dots denote normal ordering and $|0>$ is the correlated ground state. Since we consider only symmetric nuclear matter the matrix $\widehat{F}(p)$ is diagonal and degenerate with respect to isospin indices.

By using the procedures outlined in Ref. [7], the following equation for $\widehat{F}(p)$ can be derived from Eqs. (2.1)

$$
\begin{aligned}
&[p_{\ \lambda}\gamma^\lambda + \mu\gamma^0\widehat{F}(p)]_{\alpha\beta} - M\widehat{F}_{\alpha\beta}(p) \\
&- g_V\frac{1}{(2\pi)^4}\int d^4R\ e^{-ip\cdot R} < 0| : \bar{\psi}_\beta(\frac{R}{2})\gamma^\lambda_{\alpha\delta}\psi_\delta(-\frac{R}{2})V_\lambda(-\frac{R}{2}) : |0> \\
&- g_\varrho\frac{1}{(2\pi)^4}\int d^4R\ e^{-ip\cdot R} < 0| : \bar{\psi}_\beta(\frac{R}{2})[\gamma^\lambda\boldsymbol{\tau}]_{\alpha\delta}\psi_\delta(-\frac{R}{2})\cdot\mathbf{B}_\lambda(-\frac{R}{2}) : |0> \\
&+ g_S\frac{1}{(2\pi)^4}\int d^4R\ e^{-ip\cdot R} < 0| : \bar{\psi}_\beta(\frac{R}{2})\psi_\alpha(-\frac{R}{2})\Phi(-\frac{R}{2}) : |0>= 0\ . \qquad (2.3)
\end{aligned}
$$

Following Refs. [7] and [8] we neglect derivative terms in Eqs. (2.1b,c,d) so that the meson field operators are simply given by

$$
\begin{aligned}
\Phi(x) &= \frac{g_S}{m_S^2}\bar{\psi}(x)\psi(x)\ , \\
V^\lambda(x) &= \frac{g_V}{m_V^2}\bar{\psi}(x)\gamma^\lambda\psi(x)\ , \\
\mathbf{B}^\lambda(x) &= \frac{g_\varrho}{m_\varrho^2}\ \bar{\psi}(x)\gamma^\lambda\boldsymbol{\tau}\psi(x)\ . \qquad (2.4)
\end{aligned}
$$

This approximation simplifies calculations considerably, however it neglects retardation and finite–range effects in the exchange of mesons between nucleons. Nevertheless, because of the small Compton wavelengths of the heavy mesons with respect to the internucleon spacing in ordinary nuclear matter, the approximation (2.4) appears to be reasonable. Clearly, for pions this approximation would not be justified.

After substituting in Eq. (2.3) the expressions (2.4) for the meson field operators we obtain an equation which contains expectation values of products of four nucleon field operators:

$$
< 0| : \bar{\psi}_\beta(\frac{R}{2})\psi_\alpha(-\frac{R}{2})\bar{\psi}_\gamma(-\frac{R}{2})\psi_\delta(-\frac{R}{2}) : |0>\ \ . \qquad (2.5)
$$

Following Gorkov [6] these quantities are approximated by two–fold products of expectation values:

$$
\begin{aligned}
&< 0| : \bar{\psi}_\beta(\frac{R}{2})\psi_\alpha(-\frac{R}{2})\bar{\psi}_\gamma(-\frac{R}{2})\psi_\delta(-\frac{R}{2}) : |0>= \\
&< 0| : \bar{\psi}_\beta(\frac{R}{2})\psi_\alpha(-\frac{R}{2}) : |0>< 0| : \bar{\psi}_\gamma(-\frac{R}{2})\psi_\delta(-\frac{R}{2}) : |0> \\
&-\ < 0| : \bar{\psi}_\beta(\frac{R}{2})\psi_\delta(-\frac{R}{2}) : |0>< 0| : \bar{\psi}_\gamma(-\frac{R}{2})\psi_\alpha(-\frac{R}{2}) : |0> \\
&+\ < 0| : \bar{\psi}_\beta(\frac{R}{2})\bar{\psi}_\gamma(-\frac{R}{2}) : |0>< 0| : \psi_\delta(-\frac{R}{2})\psi_\alpha(-\frac{R}{2}) : |0>\ \ . \qquad (2.6)
\end{aligned}
$$

The first two terms correspond to the Hartree–Fock field. Beside these terms, Eq. (2.6) contains the product of elements of the anomalous density matrix. A nonvanishing value of this product implies the presence of the superfluid phase.

Introducing the pair amplitude

$$\widehat{D}_{\alpha\beta}(p) = \frac{1}{(2\pi)^4}\int d^4R\; e^{-ip\cdot R} < 0| : \bar{\psi}_\beta(\frac{R}{2})\bar{\psi}_\alpha(-\frac{R}{2}) : |0> \quad , \tag{2.7}$$

and the pairing field

$$\widehat{\Delta}_{\alpha\beta} = \int d^4p\; \widehat{D}_{\alpha\beta}(p) =< 0| : \bar{\psi}_\beta(x)\bar{\psi}_\alpha(x) : |0> \quad , \tag{2.8}$$

we obtain the following equation for the one–particle density matrix

$$\begin{aligned} &(\; p_\lambda\gamma^\lambda - \tilde{f}_V\rho_B\gamma^0 + \mu\gamma^0)\widehat{F}(p) - (M - \tilde{f}_S\rho_S)\widehat{F}(p) \\ &- f_V\gamma^\lambda\bar{\widehat{\Delta}}\gamma_\lambda^T\widehat{D}(p) - f_\varrho\gamma^\lambda\boldsymbol{\tau}\bar{\widehat{\Delta}}\cdot\boldsymbol{\tau}^T\gamma_\lambda^T\widehat{D}(p) + f_S\bar{\widehat{\Delta}}\widehat{D}(p) = 0 \;, \end{aligned} \tag{2.9}$$

where the effective coupling constants, $\tilde{f}_S$ and $\tilde{f}_V$, are given by the combinations

$$\tilde{f}_S = \frac{7}{8}f_S + \frac{1}{2}f_V + \frac{3}{2}f_\varrho \;, \qquad \tilde{f}_V = \frac{1}{8}f_S + \frac{5}{4}f_V + \frac{3}{4}f_\varrho \;, \tag{2.10}$$

with $f_S = (\; g_S/m_S\;)^2$, $f_V = (\; g_V/m_V\;)^2$ and $f_\varrho = (\; g_\varrho/m_\varrho\;)^2$. The matrix $\bar{\widehat{\Delta}}$ in (2.9)is conjugate to the pairing field:

$$\bar{\widehat{\Delta}}_{\alpha\beta} = \;(\gamma^0\widehat{\Delta}^\dagger\gamma^0)_{\alpha\beta} = \; < 0| : \psi_\beta(x)\psi_\alpha(x) : |0> \;. \tag{2.11}$$

The scalar density ρ_S and the baryon density ρ_B are given by

$$\begin{aligned} \rho_S &= 2Tr\int d^4p\,\widehat{F}(p)\;, \\ \rho_B &= 2Tr\int d^4p\,\gamma^0\widehat{F}(p)\;, \end{aligned} \tag{2.12}$$

with the factor 2 coming from isospin degeneracy; the traces are taken only over the spin states.

By repeating the same procedure that leads to Eq. (2.9), we obtain for the pair amplitude $\widehat{D}(p)$ an equation coupled to Eq. (2.9), that reads

$$
\begin{aligned}
(\; & p_\lambda\gamma^\lambda + \tilde{f}_V\rho_B\gamma^0 - \mu\gamma^0)\widehat{D}(p) - (M - \tilde{f}_S\rho_S)\widehat{D}(p) \\
& - f_V\gamma^\lambda\widehat{\Delta}\gamma_\lambda^T\widehat{F}(p) - f_\varrho\gamma^\lambda\boldsymbol{\tau}\widehat{\Delta}\cdot\boldsymbol{\tau}^T\gamma_\lambda^T\widehat{F}(p) + f_S\widehat{\Delta}\widehat{F}(p) = 0 \;, \qquad (2.13)
\end{aligned}
$$

In Eqs. (2.9) and (2.13) the exchange contributions to the mean field have been taken into account through the definition of the effective coupling constants of Eq. (2.10).

Before turning our attention to Eqs. (2.9) and (2.13), we discuss the symmetry and tensor properties of the pairing field. Here we consider only ${}^1S_0$ pairing of nucleons in symmetric nuclear matter. In this case the pairing field $\widehat{\Delta}$ is symmetric and degenerate with respect the isospin indices. Therefore we can consider only the isoscalar component of $\widehat{\Delta}$ and hence omit the isospin variables. Now $\widehat{\Delta}$ is a 4×4 matrix in spin space like the matrices $\widehat{F}(p)$ and $\widehat{D}(p)$. In spin space the matrix $\widehat{\Delta}$ is antisymmetric, hence it can be decomposed as

$$
\widehat{\Delta} = \; \Delta_S\sigma^{13} + \Delta_{PS}\sigma^{02} + \Delta_0\gamma^5\gamma^2 + \Delta_1\gamma^3 + \Delta_2\gamma^5\gamma^0 + \Delta_3\gamma^1 \;, \qquad (2.14)
$$

the subscripts are chosen according to the tensor properties of the various terms. The transformations properties of $\widehat{\Delta}$ can be derived from the basic transformation law of the field operator $\psi(x)$. In detail, under infinitesimal Lorentz transformations ($\Lambda^{\lambda\nu} = g^{\lambda\nu} + \epsilon^{\lambda\nu}$) the field $\widehat{\Delta}$ transforms according to

$$
\widehat{\Delta}' = \; \widehat{\Delta} + \frac{i}{4}\epsilon^{\lambda\nu}(\widehat{\Delta}\sigma_{\lambda\nu} + {\sigma_{\lambda\nu}}^T\widehat{\Delta}) \;,
$$

while for space inversion

$$
\widehat{\Delta}' = \; \gamma^0\widehat{\Delta}\gamma^0 \;.
$$

From these equations we can see that the four quantities $(\widehat{\Delta}_0, -i\widehat{\Delta}_1, \widehat{\Delta}_2, i\widehat{\Delta}_3)$ represent a four–vector and the remaining components $\widehat{\Delta}_S$ and $\widehat{\Delta}_{PS}$ are scalar and pseudoscalar quantities, respectively. Since we are considering a homogeneous system at rest, only the components Δ_S and Δ_0 in Eq. (2.14) can differ from zero:

$$
\widehat{\Delta} = \; \Delta_S\sigma^{13} + \Delta_0\gamma^5\gamma^2 \;. \qquad (2.15)
$$

Moreover, because of the invariance of the equilibrium state under time–reversal the components Δ_S and Δ_0 can be assumed to be real and the matrix $\hat{\widehat{\Delta}}$ of Eq. (2.11) becomes

$$\hat{\hat{\Delta}} = \hat{\Delta}\ . \tag{2.16}$$

From Eqs. (2.9) and (2.13), after some algebraic manipulations, we obtain two separate equations for the density matrix $\widehat{F}(p)$ and for the pair amplitude $\widehat{D}(p)$:

$$\left(R^\mu(p)\gamma_\mu - \overline{M}(p)\right)\widehat{F}(p) = 0 \tag{2.17}$$

and

$$\left(R^\mu(-p)\gamma_\mu^T - \overline{M}(-p)\right)\widehat{D}(p) = 0\ . \tag{2.18}$$

The components of the four–vector $R_\mu(p)$ are

$$\begin{aligned} \mathbf{R} &= \mathbf{p}\ , \\ R_0(p) &= p_0 + \tilde{\mu} + \frac{2}{\mathcal{K}(p)}\left(g^2\Delta_S^2\tilde{\mu} - f^2\Delta_0^2 p_0 + fg\Delta_S\Delta_0 M^*\right)\ , \end{aligned} \tag{2.19}$$

and the mass term $\overline{M}(p)$ is given by

$$\overline{M}(p) = M^* - \frac{2}{\mathcal{K}(p)}\left(f^2\Delta_0^2\, M^* - f\, g\Delta_0\, \Delta_S(p_0 - \tilde{\mu})\right)\ , \tag{2.20}$$

Here the coupling constants g and f are given by the combinations

$$g = f_S - 4\,(f_V + f - \varrho)\ , \qquad f = f_S + 2\,(f_V + f_\varrho)\ , \tag{2.21}$$

and the quantity $\mathcal{K}(p)$ is expressed as

$$\mathcal{K}(p) = (p_0 - \tilde{\mu})^2 - E_{\mathbf{p}}^2 + f^2\,\Delta_0^2 - g^2\,\Delta_S^2\ , \tag{2.22}$$

where $E_{\mathbf{p}} = (\mathbf{p}^2 + M^{*2})^{1/2}$, while $\tilde{\mu} = \mu - \tilde{f}_V\rho_B$ is the effective chemical potential and $M^* = M - \tilde{f}_S\rho_S$ is the effective nucleon mass.

Equation (2.17) tells us that the matrix $\widehat{F}(p)$ can be put in the form

$$\widehat{F}(p) = F(p) + \gamma^\mu F_\mu(p)\ , \tag{2.23a}$$

with

$$F_\mu(p) = \frac{R_\mu(p)}{\overline{M}(p)} F(p) \ , \tag{2.23b}$$

and $F(p)$ a scalar quantity.

Equations (2.17) and (2.18) contain the components of the pairing field as parameters, these components must be determined self–consistently. This can be done by solving Eq. (2.13) with respect to $\widehat{D}(p)$ with the aid of Eqs. (2.23). For the two nonvanishing components of the pair amplitude

$$\widehat{D}(p) = D_S(p)\sigma^{13} + D_0(p)\gamma^5\gamma^2$$

we obtain

$$D_S(p) = \frac{2}{\mathcal{K}(p)} \left(g\Delta_S\, \tilde{\mu} F_0(p) - f\Delta_0(p_0 F(p) - M^* F_0(p))\right) \ , \tag{2.24a}$$

$$D_0(p) = \frac{2}{\mathcal{K}(p)} \left(g\Delta_S\, \tilde{\mu} F(p) - f\Delta_0(p_0 F_0(p) - M^* F(p))\right) \ . \tag{2.24b}$$

Now we derive the energy spectrum using our approach. Substituting in Eq. (2.17) the formal solution given by Eqs. (2.23), we can see that the components of the four–vector $R_\mu(p)$ must satisfy the constraint

$$R_\mu R^\mu = \overline{M}^2(p) \ .$$

This equation, for a fixed $\mathbf{p}$, determine the allowed values of p_0

$$\begin{aligned} p_0^2 = \ & E_{\mathbf{p}}^2 + \tilde{\mu}^2 + g^2\Delta_S^2 + f^2\Delta_0^2 \ \pm \\ & 2\left(\tilde{\mu}^2 E_{\mathbf{p}}^2 + g^2 f^2 \Delta_S^2\Delta_0^2 + f^2\Delta_0^2\mathbf{p}^2 - 2fg\,\Delta_S\Delta_0\,\tilde{\mu} M^*\right)^{1/2} \ , \end{aligned} \tag{2.25}$$

which correspond to the energies of elementary excitations referred to the effective chemical potential $\tilde{\mu}$. The upper sign refers to elementary excitations of the Dirac sea. Actually in the absence of the pairing field the energies of these excitations become

$$p_0 = (E_{\mathbf{p}} + \tilde{\mu}) \ .$$

We neglect contributions from the Dirac sea and consider only the energy value given by the lower sign in Eq. (2.25). This defines the energy $\mathcal{E}_{\mathbf{p}}$ of a quasiparticle with momentum $\mathbf{p}$ in the superfluid phase.

We turn now to the explicit evaluation of the matrices $\widehat{F}(p)$ and $\widehat{D}(p)$. By inserting a complete set of energy eigenvectors in Eq. (2.2) we have

$$\widehat{F}_{\alpha\beta}(p) = \frac{1}{(2\pi)^3} \int d^3\mathbf{R}\; e^{i\mathbf{p}\cdot\mathbf{R}} \sum_n \delta(p_0 + E_n) < 0|\bar{\psi}_\beta(\frac{\mathbf{R}}{2})|n >< n|\psi_\alpha(-\frac{\mathbf{R}}{2})|0 > \quad , \tag{2.26}$$

where $|n>$ represents a quasiparticle state $|n>\equiv|\mathcal{E}_{\mathbf{p}_n}, \mathbf{p}_n, \lambda_n>$ with spin label λ_n. This equation, toghether with Eq. (2.17), shows that the one–particle density matrix $\widehat{F}(p)$ can be put in the form

$$\widehat{F}_{\alpha\beta}(p) = \frac{1}{(2\pi)^3} \sum_\lambda g^*_\lambda(\mathbf{p}) \bar{u}^{(\beta)}_\lambda(\mathbf{p}) u^{(\alpha)}_\lambda(\mathbf{p}) g_\lambda(\mathbf{p}) \delta(p_0 + \mathcal{E}_{\mathbf{p}}) \ , \tag{2.27}$$

where the spinor $u_\lambda(\mathbf{p})$ obeys the equation

$$(R_\mu(\mathbf{p}, -\mathcal{E}_{\mathbf{p}})\gamma^\mu - \overline{M}(-\mathcal{E}_{\mathbf{p}}))u_\lambda(\mathbf{p}) = \ 0 \ . \tag{2.28}$$

With the normalization $u^*_\lambda(\mathbf{p})u_\lambda(\mathbf{p}) = 1$, the quantity $g_\lambda(\mathbf{p})$ represents the probability amplitude of finding a hole with momentum $-\mathbf{p}$ and spin label $-\lambda$ in the quasiparticle state $|\mathcal{E}_{\mathbf{p}}, \mathbf{p}, \lambda>$.

For the pair amplitude $\widehat{D}(p)$, with the same procedure and with the aid of Eq. (2.18), we obtain the expression

$$\widehat{D}_{\alpha\beta}(p) = \frac{1}{(2\pi)^3} \sum_\lambda (-1)^\lambda g^*_{-\lambda}(\mathbf{p}) \bar{u}^{(\beta)}_\lambda(\mathbf{p}) \bar{v}^{(\alpha)}_\lambda(\mathbf{p}) \tilde{g}_\lambda(\mathbf{p}) \delta(p_0 + \mathcal{E}_{\mathbf{p}}) \ . \tag{2.29}$$

The equation for the spinor $v_\lambda(\mathbf{p})$ can be derived from Eq. (2.18) and reads

$$(R_\mu(-\mathbf{p}, \mathcal{E}_{\mathbf{p}})\gamma^\mu - \overline{M}(\mathcal{E}_{\mathbf{p}}))v_\lambda(\mathbf{p}) = \ 0 \ . \tag{2.30}$$

We choose for $v_\lambda(\mathbf{p})$ the same normalization as $u_\lambda(\mathbf{p})$. In Eq. (2.30) the quantity $\tilde{g}_\lambda(\mathbf{p})$ is the probability amplitude of finding a particle with momentum $\mathbf{p}$ and spin label λ in the quasiparticle state $|\mathcal{E}_{\mathbf{p}}, \mathbf{p}, \lambda>$. With the choice of phase made in Eq. (2.29) the product

$g^*_{-\lambda}(\mathbf{p})\tilde{g}_\lambda(\mathbf{p})$ is real and independent of λ, as can be seen by substituting in Eq. (2.13) the expression (2.27) for $\widehat{F}(p)$.

It is implicit in our approach that a quasiparticle state is a superposition of one–particle and one–hole states, so that the following normalization condition holds

$$|g_\lambda(\mathbf{p})|^2 + |\tilde{g}_\lambda(\mathbf{p})|^2 = 1 \ . \tag{2.31}$$

Moreover, since we are considering a homogeneous and isotropic system, $|g_\lambda(\mathbf{p})|$ and $|\tilde{g}_\lambda(\mathbf{p})|$ do not depend on λ.

The specific form of the spinors $u_\lambda(\mathbf{p})$ and $v_\lambda(\mathbf{p})$ is determined by the signs of $R_0(p)$ and $\overline{M}(p)$. Explicit calculations show that $R_0(\pm\mathcal{E}_\mathbf{p})$ remains positive for any value of $|\mathbf{p}|$, whereas the mass term $\overline{M}(-\mathcal{E}_\mathbf{p})$ of Eq. (2.28) has a peculiar behaviour. It is a monotonically decreasing function of $|\mathbf{p}|$, it takes negative values for $|\mathbf{p}|$ larger than a certain value p_c and becomes infinitely negative at a finite value of $|\mathbf{p}| > p_c$. The mass term $\overline{M}(\mathcal{E}_\mathbf{p})$ of Eq. (2.30) instead is always positive and almost constant.

For $|\mathbf{p}| \leq p_c$ the solutions of Eq. (2.28) are given by

$$u_\lambda(\mathbf{p}) = \Big[\frac{R_0(-\mathcal{E}_\mathbf{p}) + \overline{M}(-\mathcal{E}_\mathbf{p})}{2R_0(-\mathcal{E}_\mathbf{p})}\Big]^{1/2} \begin{pmatrix} \chi_\lambda \\ \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{R_0(-\mathcal{E}_\mathbf{p}) + \overline{M}(-\mathcal{E}_\mathbf{p})}\,\chi_\lambda \end{pmatrix} , \tag{2.32a}$$

whereas for $|\mathbf{p}| > p_c$ we have to choose the spinors

$$u_\lambda(\mathbf{p}) = \Big[\frac{R_0(-\mathcal{E}_\mathbf{p}) + |\overline{M}(-\mathcal{E}_\mathbf{p})|}{2R_0(-\mathcal{E}_\mathbf{p})}\Big]^{1/2} \begin{pmatrix} \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{R_0(-\mathcal{E}_\mathbf{p}) + |\overline{M}(-\mathcal{E}_\mathbf{p})|}\,\chi_\lambda \\ \\ \chi_\lambda \end{pmatrix} , \tag{2.32b}$$

as solutions of Eq. (2.28). The solutions of Eq. (2.30) are given by the spinors

$$v_\lambda(\mathbf{p}) = \Big[\frac{R_0(\mathcal{E}_\mathbf{p}) + \overline{M}(\mathcal{E}_\mathbf{p})}{2R_0(\mathcal{E}_\mathbf{p})}\Big]^{1/2} \begin{pmatrix} \chi_\lambda \\ \\ -\dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{R_0(\mathcal{E}_\mathbf{p}) + \overline{M}(\mathcal{E}_\mathbf{p})}\,\chi_\lambda \end{pmatrix} . \tag{2.33}$$

In equations above χ_λ denote the usual two–component Pauli spinors.

We remark that the energy spectrum of excitations of the Dirac sea remains well separated from the quasiparticle spectrum. In fact the mass terms in Eqs. (2.17) and (2.18) for the antiparticle case, are always positive for any value of $|\mathbf{p}|$.

By substituting the spinors (2.32b) and (2.33) in Eq. (2.29), we can see that for $|\mathbf{p}| > p_c$, the two components

$$D_S(\mathbf{p}) = \frac{1}{4} Tr \sigma^{13} \widehat{D}(\mathbf{p})$$

and

$$D_0(\mathbf{p}) = \frac{1}{4} Tr \gamma^5 \gamma^2 \widehat{D}(\mathbf{p})$$

vanish. This implies that $F_0(\mathbf{p})$ also vanishes for $|\mathbf{p}| > p_c$, see Eqs. (2.24). The occupation number of particles in the correlated ground state displays a discontinuity.

From Eqs (2.24) with the aid of the normalization condition (2.31), we can determine the amplitudes $g_\lambda(\mathbf{p})$ and $\tilde{g}_\lambda(\mathbf{p})$, and the matrices $\widehat{F}(p)$ and $\widehat{D}(p)$, as functions of the parameters M^*, $\tilde{\mu}$, Δ_S and Δ_0. These parameters can be calculated by solving the four coupled equations that are obtained by fixing the baryon density

$$\frac{2}{3} \frac{1}{\pi^2} p_F^3 = 8 \int d\mathbf{p} F_0(\mathbf{p}) \ , \tag{2.34a}$$

and using the self–consistency relations for the effective nucleon mass

$$M^* = M - 8 \tilde{f}_S \int d\mathbf{p} \frac{\overline{M}(-\mathcal{E}_\mathbf{p})}{R_0(-\mathcal{E}_\mathbf{p})} F_0(\mathbf{p}) \tag{2.34b}$$

and for the components of the pairing field

$$\Delta_S = 2 \int \frac{d\mathbf{p}}{\mathcal{K}(\mathbf{p}, -\mathcal{E}_\mathbf{p})} \left(g \Delta_S \tilde{\mu} F_0(\mathbf{p}) - f \Delta_0 (-\mathcal{E}_\mathbf{p} F(\mathbf{p}) - M^* F_0(\mathbf{p})) \right) \ , \tag{2.34c}$$

$$\Delta_0 = 2 \int \frac{d\mathbf{p}}{\mathcal{K}(\mathbf{p}, -\mathcal{E}_\mathbf{p})} \left(g \Delta_S \tilde{\mu} F(\mathbf{p}) - f \Delta_0 (-\mathcal{E}_\mathbf{p} F_0(\mathbf{p}) - M^* F(\mathbf{p})) \right) \ . \tag{2.34d}$$

The momentum p_c, for which $F_0(\mathbf{p})$ vanishes, plays the role of an effective cutoff in the integrals (2.34). This fact avoids introducing an artificial cutoff to make the integrals

(2.34c) and (2.34d) converge. The dependence of p_c on Δ_S and Δ_0 amounts to a further self–consistency relation.

Equations (2.34c) and (2.34d) replace the gap equation of the nonrelativistic case. In the nonrelativistic limit $F(\mathbf{p}) \to F_0(\mathbf{p})$, hence these two equations become identical and the two components of the pairing field coincide: $\Delta_S = \Delta_0$. Moreover, the quasiparticle energy $\mathcal{E}_{\mathbf{p}}$ acquires the usual expression of the BCS theory

$$\mathcal{E}_{\mathbf{p}} = \ ((E_{\mathbf{p}} - \tilde{\mu})^2 + (f+g)^2\Delta^2)^{1/2} \ ,$$

where $\Delta = \Delta_S = \Delta_0$.

III. RESULTS

In this section we investigate the solutions of Eqs. (2.34). These equations have been derived by starting from the assumptions (2.4), which neglect the finite range of the nucleon–nucleon interaction.

Concerning the coupling constants f_V and f_S we choose their value so as to reproduce the binding energy (15.75 MeV) of saturated nuclear matter with a Fermi momentum of 1.42 fm^{-1} (see Ref. [7]). These values are:

$$f_S = 2.37{\cdot}10^{-4} MeV^{-2}, \quad f_V = 1.45{\cdot}10^{-4} MeV^{-2} \ .$$

For the coupling constant f_ϱ we have taken the value determined by the $\varrho \to 2\pi$ decay

$$f_\varrho = 1.55{\cdot}10^{-5} MeV^{-2} \ .$$

Then the effective constants f and g for the pairing field are

$$f = 5.58{\cdot}10^{-4} MeV^{-2}, \quad g = -4.05{\cdot}10^{-4} MeV^{-2} \ .$$

The relevant quantities for the quasiparticle energy spectrum are $f\,\Delta_0$ and $g\,\Delta_S$. In Fig. 1 these quantities are displayed, together with their sum, as functions of the Fermi momentum. The sum $f\,\Delta_0 + g\,\Delta_S$ approximatively reproduces the gap in the quasiparticle

energy spectrum. Figure 1 shows that a superfluid solution of Eqs. (2.34) is present in the range of densities corresponding to $p_F < 1.35\,fm^{-1}$. Moreover one can see that $f\,\Delta_0$ and $g\,\Delta_S$ separately are very large with respect to their sum, i.e. the gap in the quasiparticle spectrum is determined by difference between two large and not very different numbers.

In Fig. 2 the excitation spectra of the superfluid and normal phases are displayed as functions of $|\mathbf{p}|$ for $p_F = 0.9\,fm^{-1}$. A numerical analysis shows that for $|\mathbf{p}| >\sim 1.9\,fm^{-1}$ the quasiparticle energies lie slightly below the excitation energies of the normal phase, $E_{\mathbf{p}} - \tilde{\mu}$. This fact gives rise to the cutoff in the integrals (2.34). Since for a sufficiently high value of $|\mathbf{p}|$ the quantity $\mathcal{K}(p)$ of Eq. (2.22) may vanish, the second term of $\overline{M}(p)$, Eq. (2.20), which is positive, can become larger than M^*. The cutoff p_c is given by the value of $|\mathbf{p}|$ for which the r.h.s of Eq. (2.20) vanishes.

In Fig. 3 we show both the sum $f\,\Delta_0 + g\,\Delta_S$ and the energy gap as a function of the Fermi momentum. We can see that, though remaining small, the difference between these two quantities increases with p_F. This is because relativistic effects become more important with increasing density. In the non relativistic limit the energy gap and the sum $f\,\Delta_0 + g\,\Delta_S$ coincide. Then, the difference between these two quantities could give an insight about the relevance of relativistic effects. In the region around $p_F = 0.9\,fm^{-1}$ where the energy gap takes its maximum value, this difference is not very large, only about 10%. However we remark that the occurrence of a cutoff in the integrals for the pairing field is obtained only using a relativistic expression (Eq. (2.25)) for the quasiparticle energy.

Figure 4 shows that the pair amplitude is rather sharply peaked about the Fermi momentum. This important feature is the basis of the approximation, that we use in the next section to take into account the finite range of the nucleon–nucleon interaction.

For the two values of the Fermi momentum for which superfluidity disappears and the energy gap becomes maximum, our results qualitatively agree with previous treatments, both nonrelativistic [2,9–14] and relativistic [3,5]. Instead for the most relevant quantity of the superfluid phase, the energy gap, our calculations yield values that are twice the generally accepted estimates. The value of the sum $f + g$, which plays the role of an effective coupling

constant for interacting paired nucleons is too large. It is worth noticing that the role of $f+g$ in determining the energy gap is enhanced by a cooperative effect due to the self–cosistency constraint for the cutoff p_c: if the gap becomes larger the value of p_c increases, then the contribution to the pairing field of the integrals (2.34c) and (2.34d) also increases, giving rise to a larger gap.

IV. FINITE–RANGE EFFECTS

In this section we evaluate effects due to the finite range of the nucleon–nucleon interaction. For the effects of retardation in the propagation of the meson fields we will give only an estimate of their magnitude and argue that these effects do not play an important role. For simplicity we take into account the finite range of the effective interaction between paired nucleons only when calculating quantities that are relevant to the superfluid phase. In deriving the Hartree–Fock field, instead, we retain the approximation of Eqs. (2.4). Thus for quantities containing the pairing field, instead of the approximated expressions (2.4) we introduce the formal solutions of Eqs. (2.1b,c,d):

$$\begin{aligned}
\Phi(x) &= g_S \int d^4y\, \mathcal{D}(x-y)\bar{\psi}(y)\psi(y)\ , \\
V_\lambda(x) &= g_V \int d^4y\, \mathcal{D}^{(\omega)}_{\lambda\mu}(x-y)\bar{\psi}(y)\gamma^\mu\psi(y)\ , \\
\mathbf{B}_\lambda(x) &= g_\varrho \int d^4y\, \mathcal{D}^{(\varrho)}_{\lambda\mu}(x-y)\bar{\psi}(y)\gamma^\mu\boldsymbol{\tau}\psi(y)\ \ ,
\end{aligned} \tag{4.1}$$

where $\mathcal{D}(x-y)$, $\mathcal{D}^{(\omega)}_{\lambda\mu}(x-y)$ and $\mathcal{D}^{(\varrho)}_{\lambda\mu}(x-y)$ are the propagators of the meson fields.

We explicitly derive the equations for the contributions of the scalar field alone, for the vector fields only the final results are reported.

In Eq. (2.6) the term containing elements of the anomalous density matrix acquires the form:

$$\int d^4y\, \mathcal{D}(y-\frac{R}{2}) < 0| : \bar{\psi}_\beta(\frac{R}{2})\bar{\psi}_\gamma(y) : |0 >< 0| : \psi_\gamma(y)\psi_\alpha(-\frac{R}{2}) : |0 >\ \ ,$$

then in Eq. (2.9) the term $f_S\hat{\bar{\Delta}}\,\widehat{D}(p)$ is replaced by

$$g_S^2 \int d^4q\, \mathcal{D}(p-q)\gamma^0 \widehat{D}^\dagger(q)\gamma^0\, \widehat{D}(p)\ . \tag{4.2}$$

Here $\mathcal{D}(p-q)$ is the Fourier transform of the propagator $\mathcal{D}(x-y)$.

Analogously, in Eq. (2.13) we have to make the substitution

$$f_S\widehat{\Delta}\,\widehat{F}(p) \to\ g_S^2 \int d^4q\, \mathcal{D}(p-q)\widehat{D}(q)\,\widehat{F}(p)\ . \tag{4.3}$$

Assuming that retardation effects are negligible, we can put $p_0 - q_0 = 0$ in evaluating the integrals of Eqs. (4.2) and (4.3). Thus these integrals depend only on $\mathbf{p}$. This fact greatly simplifies calculations. In particular, the quasiparticle energies can still be expressed in closed form; it is sufficient to replace in Eq. (2.25) the terms $g\Delta_S$ and $f\Delta_0$ with the analogous $\mathbf{p}$–dependent quantities.

We introduce now a further and more fundamental approximation. In the previous section we have shown that the components of the pair amplitude $\widehat{D}(q)$ are strongly peaked about p_F. The width of this peak is much smaller than the range over which the meson propagators present an appreciable variation. In fact this range is typically of order $\sim m_S, m_V, m_\varrho$. For this reason we expect that the values of the integrals (4.2) and (4.3) can be given by

$$g_S^2\,\overline{\mathcal{D}}(|\mathbf{p}|, p_F) \int d^4q\, \gamma^0\widehat{D}^\dagger(q)\gamma^0 \;=\; g_S^2\,\overline{\mathcal{D}}(|\mathbf{p}|, p_F)\gamma^0\widehat{\Delta}^\dagger\gamma^0\ , \tag{4.4}$$

$$g_S^2\,\overline{\mathcal{D}}(|\mathbf{p}|, p_F) \int d^4q\, \widehat{D}(q) \;=\; g_S^2\,\overline{\mathcal{D}}(|\mathbf{p}|, p_F)\widehat{\Delta}\ , \tag{4.5}$$

with a satisfactory approximation. Here $\overline{\mathcal{D}}(|\mathbf{p}|, p_F)$ stands for the average over the directions of $\mathbf{q}$. Thus the quantity $f_S\widehat{\Delta}$ in Eqs. (2.9) and (2.13) is simply replaced by

$$f_S\, a_S(|\mathbf{p}|, p_F)\,\widehat{\Delta}\ , \tag{4.6}$$

where the factor

$$a_S(|\mathbf{p}|, p_F) \;=\; m_S^2\overline{\mathcal{D}}(|\mathbf{p}|, p_F) \;=\; \frac{m_S^2}{4p_F|\mathbf{p}|}\,\ln\!\left(\frac{m_S^2 + (|\mathbf{p}| + p_F)^2}{m_S^2 + (|\mathbf{p}| - p_F)^2}\right)$$

represents the finite–range correction to the contribution of the scalar meson.

As far as the vector mesons are concerned, with some algebraic manipulations we can see that the matrices $f_V\gamma^\lambda\widehat{\Delta}\gamma_\lambda^T$ and $f_V\gamma^\lambda\boldsymbol{\tau}\widehat{\Delta}\cdot\boldsymbol{\tau}\gamma_\lambda^T$ in Eqs. (2.9)and (2.13) must be replaced by

$$4f_V(\frac{3}{4}a_V(|\mathbf{p}|,p_F)+\frac{1}{4})\Delta_S\sigma^{13}-2f_V(\frac{1}{4}a_V(|\mathbf{p}|,p_F)+\frac{1}{2})\Delta_0\gamma^5\gamma^0 \tag{4.7a}$$

for the ω meson, and by a similar expression for the ϱ meson,

$$4f_\varrho(\frac{3}{4}a_\varrho(|\mathbf{p}|,p_F)+\frac{1}{4})\Delta_S\sigma^{13}-2f_\varrho(\frac{1}{4}a_\varrho(|\mathbf{p}|,p_F)+\frac{1}{2})\Delta_0\gamma^5\gamma^0 \ . \tag{4.7b}$$

Here

$$a_V(|\mathbf{p}|,p_F)=\ m_V^2\overline{\mathcal{D}}(|\mathbf{p}|,p_F)=\ \frac{m_V^2}{4p_F|\mathbf{p}|}\ \ln\Big(\frac{m_V^2+(|\mathbf{p}|+p_F)^2}{m_V^2+(|\mathbf{p}|-p_F)^2}\Big)\ ,$$

$$a_\varrho(|\mathbf{p}|,p_F)=\ m_\varrho^2\overline{\mathcal{D}}(|\mathbf{p}|,p_F)=\ \frac{m_\varrho^2}{4p_F|\mathbf{p}|}\ \ln\Big(\frac{m_\varrho^2+(|\mathbf{p}|+p_F)^2}{m_\varrho^2+(|\mathbf{p}|-p_F)^2}\Big)\ .$$

The correction factors are given by the bracketed terms in Eqs. (4.7).

Summarizing, in the present approximation the finite range of the nucleon–nucleon interaction is taken into account by replacing in all the equations of Sect. II the pairing coupling constants f and g, with the combinations

$$a_f(|\mathbf{p}|,p_F)=\ f_S\,a_S(|\mathbf{p}|,p_F)+f_V(a_V(|\mathbf{p}|,p_F)+1)+f_\varrho(a_\varrho(|\mathbf{p}|,p_F)+1) \tag{4.8a}$$

and

$$a_g(|\mathbf{p}|,p_F)=\ f_S\,a_S(|\mathbf{p}|,p_F)-f_V(3a_V(|\mathbf{p}|,p_F)+1)-f_\varrho(3a_\varrho(|\mathbf{p}|,p_F)+1) \tag{4.8b}$$

respectively.

The pairing coupling constants, separately considered, are not much affected by the finite–range corrections. In fact, the values of $a_f(|\mathbf{p}|,p_F)$ and $a_g(|\mathbf{p}|,p_F)$ for $|\mathbf{p}|=p_f$, with $0.6\,fm^{-1}<p_F<1.0\,fm^{-1}$, are smaller than f and g by about 10% and 2%, respectively. However, for the sum $f+g$, that practically determines the magnitude of the energy gap, the correction is more important. In the same range of p_F the value of $a_f(p_F,p_F)+a_g(p_F,p_F)$ is about $\frac{2}{3}(f+g)$.

The most important effects of the finite–range corrections are a substantial quenching of the pairing field and a reduction of the energy gap by an overall factor three or more. This is shown in Figs. 5 and 6 where the calculated components of the pairing field and of the energy gap are shown as functions of p_F. Moreover, with respect to the zero–range approximation the domain of p_F where the superfluid phase can arise, is narrowed and the maximum of the energy gap is shifted towards a lower value of p_F,

In Fig. 7 the quasiparticle energy together with the excitation spectrum of the normal phase is displayed for $p_F = 0.8 fm^{-1}$, where the energy gap takes now its maximum value. The value of $|\mathbf{p}|$ where the two curves cross, corresponds to the cutoff p_c. A numerical analysis shows that $p_c \sim 1.7 fm^{-1}$.

Finally, in Fig. 8 the components of the pair amplitude are shown as functions of $|\mathbf{p}|$ for $p_F = 0.8 fm^{-1}$. The finite–range corrections makes the pair amplitude even more peaked about p_F. This fact further justifies the approximation expressed by Eqs. (4.4) and (4.5). That approximation simplifies calculations substantially. In fact, the basic quantities, i.e. the components of the pairing field, that are determined self–consistently, are still two constants. We have assessed the reliability of this approximation by successive iterations of Eqs. (4.2) and (4.3), starting from Eqs. (4.4) and (4.5). The correction coming from the first iteration amounts to a few percent, while the second iteration does not substantially modify the first–order results.

We have evaluated also the order of magnitude of retardation effects, replacing $\mathcal{E}_{\mathbf{p}}$ and $\mathcal{E}_{\mathbf{q}}$ instead of p_0 and q_0 in Eqs. (4.2) and (4.3). The change is less than one percent.

V. SUMMARY

We have investigated in the framework of a relativistic model, the possibility for the onset of a superfluid phase in symmetric nuclear matter and the features of this phase. We have derived equations for the relevant quantities of the superfluid phase by making a relativistic generalization of the scheme introduced by Gorkov to study superconductivity in electron

systems. In this scheme nuclear matter is described as an ensemble of quasiparticles moving in the Hartree–Fock field plus a self–consistent paring field. At first both the Hartree–Fock field and the pairing field have been treated on the same foot by using an approximation where the finite range and the retardation of the meson propagation between nucleons have been neglected. Then, we have improved our approach by introducing finite–range effects for the quantities pertaining to the superfluid phase.

In the relativistic treatment of the pairing process for a system at rest the pairing field has two different components: a Lorentz scalar Δ_S and the time–component Δ_0 of a four–vector. The different behaviour of the two components under Lorentz transformations must be properly taken into account in the relativistic hydrodynamics of nuclear systems in the superfluid phase.

In the non relativistic limit Δ_S and Δ_0 coincide. In our approach they are slightly different. This does not mean, however, that relativistic effects are negligible. Actually, the expression for the quasiparticle energy derived in our calculations, differs from the nonrelativistic expression of the BCS theory in an essential way. The quasiparticle energy given by Eq. (2.25) or by its analogous expression that includes finite–range corrections, displays the salient feature that it can be less than $(E_{\mathbf{p}} - \tilde{\mu})$ beyond a certain value of $|\mathbf{p}|$. This determines the occurence of a cutoff in the relativistic gap equations (2.34c) and (2.34d), that does not appear in the analogous nonrelativistic expression. This cutoff removes the contributions of high $|\mathbf{p}|$ components of the interaction to the gap equation, thus allowing the use of nucleon–nucleon interactions that are only slowly decreasing for high values of $|\mathbf{p}|$. This fact reduces the energy gap appreciably.

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FIGURES

FIG. 1. Components of the pairing field (times the respective coupling constants) as a function of p_F. The solid line and the dashed line correspond to the vector and the scalar components, respectively. The dotted line gives the sum of these two quantities.

FIG. 2. Quasiparticle energy (solid line) and single–particle energy $E_{\mathbf{p}} - \tilde{\mu}$ (dashed line), for $p_F = 0.9 fm^{-1}$.

FIG. 3. Energy gap (solid line) together with the sum $f\Delta_0 + g\Delta_S$ (dashed line) versus p_F.

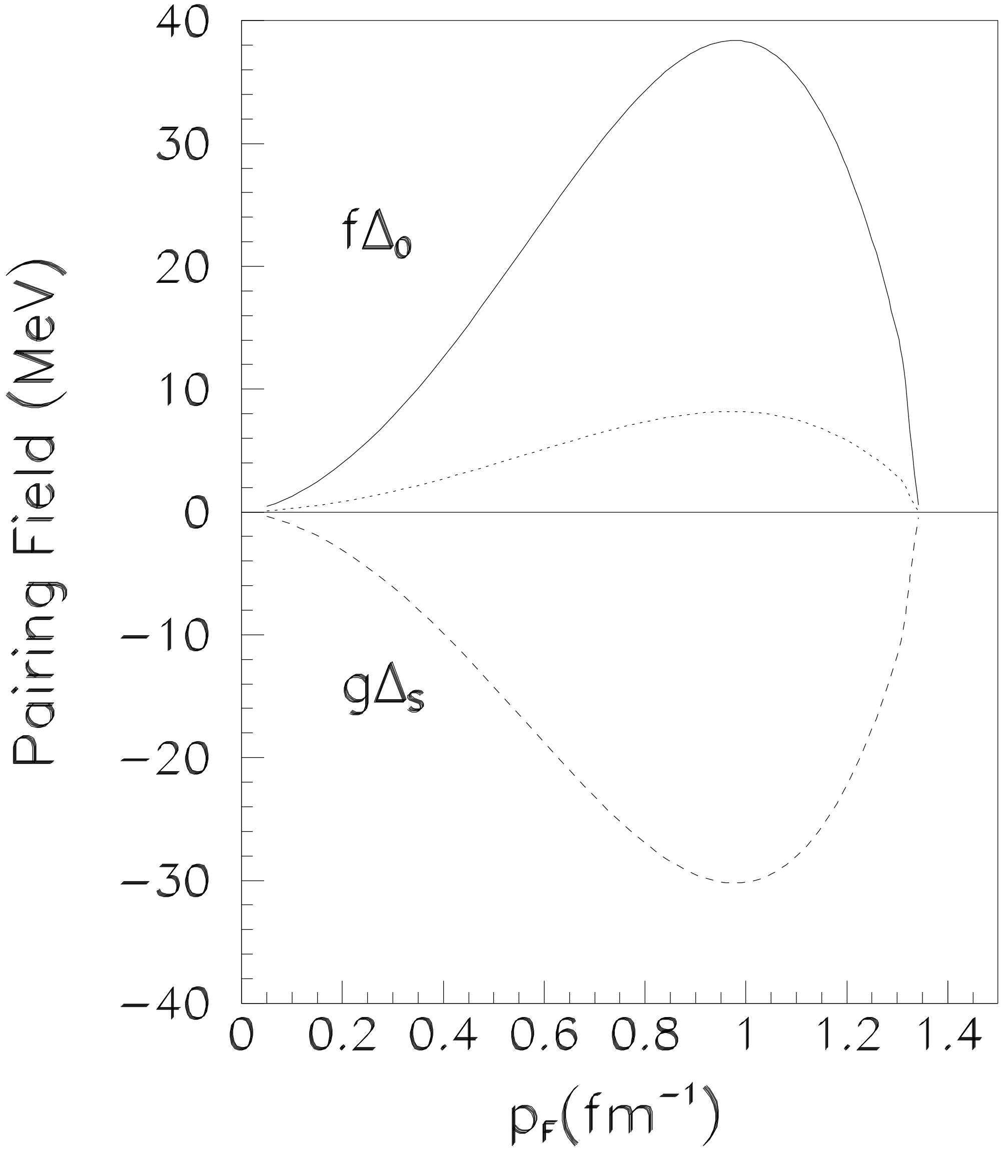
FIG. 4. Components of the pair amplitude (times $(2\pi\hbar)^3$) as a function of $|\mathbf{p}|$ for $p_F = 0.9 fm^{-1}$. The solid and the dashed lines correspond to the vector and scalar components, respectively.

FIG. 5. Components of the pairing field at the Fermi surface as functions of p_F. Finite–range corrections are included. The vector component (solid line) and the scalar component (dashed line) are respectively multiplied by a_f and a_g (see Eqs. (4.8)).

FIG. 6. Energy gap (solid line) versus p_F together with the sum $a_f\Delta_0 + a_g\Delta_S$, evaluated at the Fermi surface (dashed line). The finite–range corrections are included.

FIG. 7. Same as Fig. 2 with finite–range corrections and for $p_F = 0.8 fm^{-1}$.

FIG. 8. Same as Fig. 4 with finite–range corrections and for $p_F = 0.8 fm^{-1}$. The two components of the pair amplitude are practically indistinguishable at this density.



40
30
20
10
0
−10
−20
−30
−40
Pairing Field (MeV)
$f\Delta_0$
$g\Delta_s$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
$p_F(fm^{-1})$

Fig.1

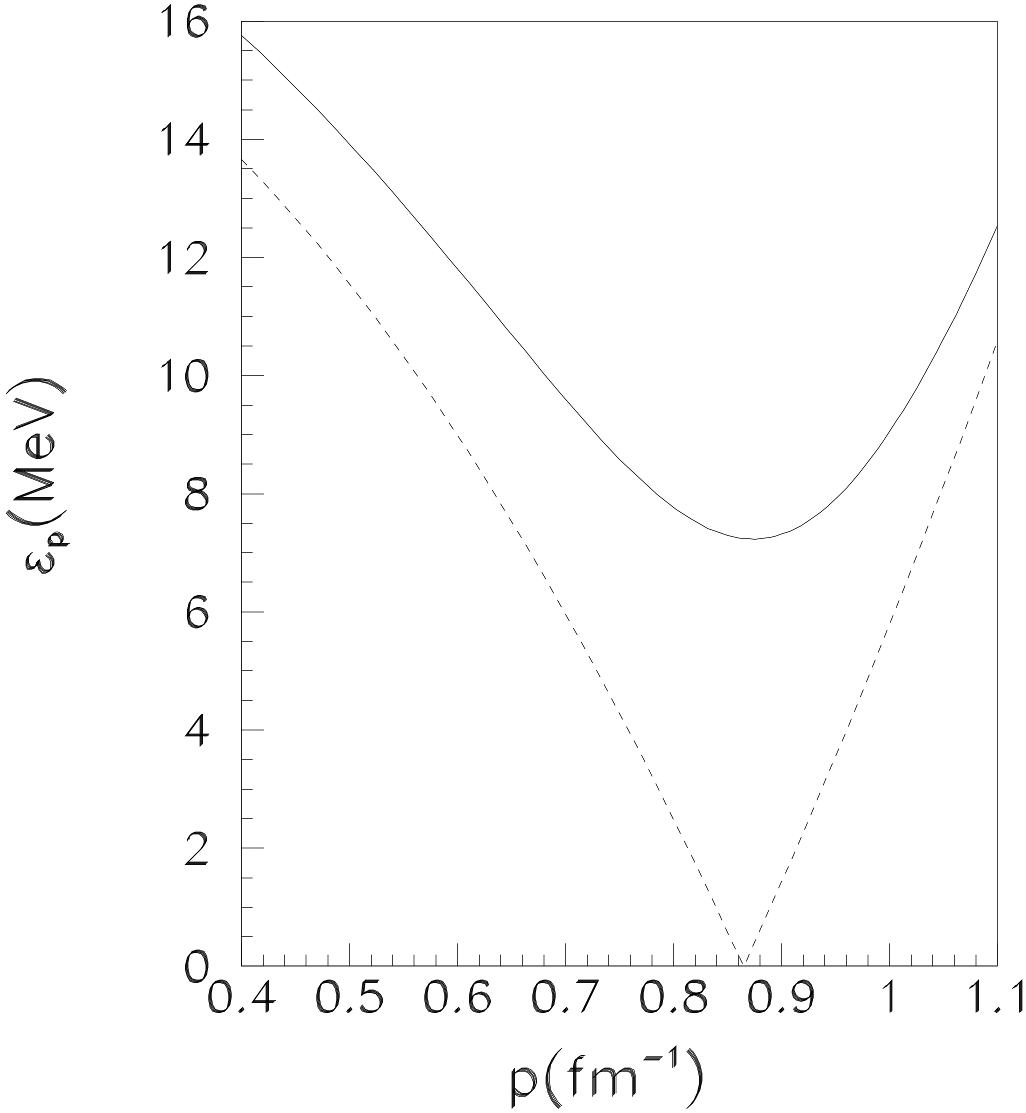


Fig.2

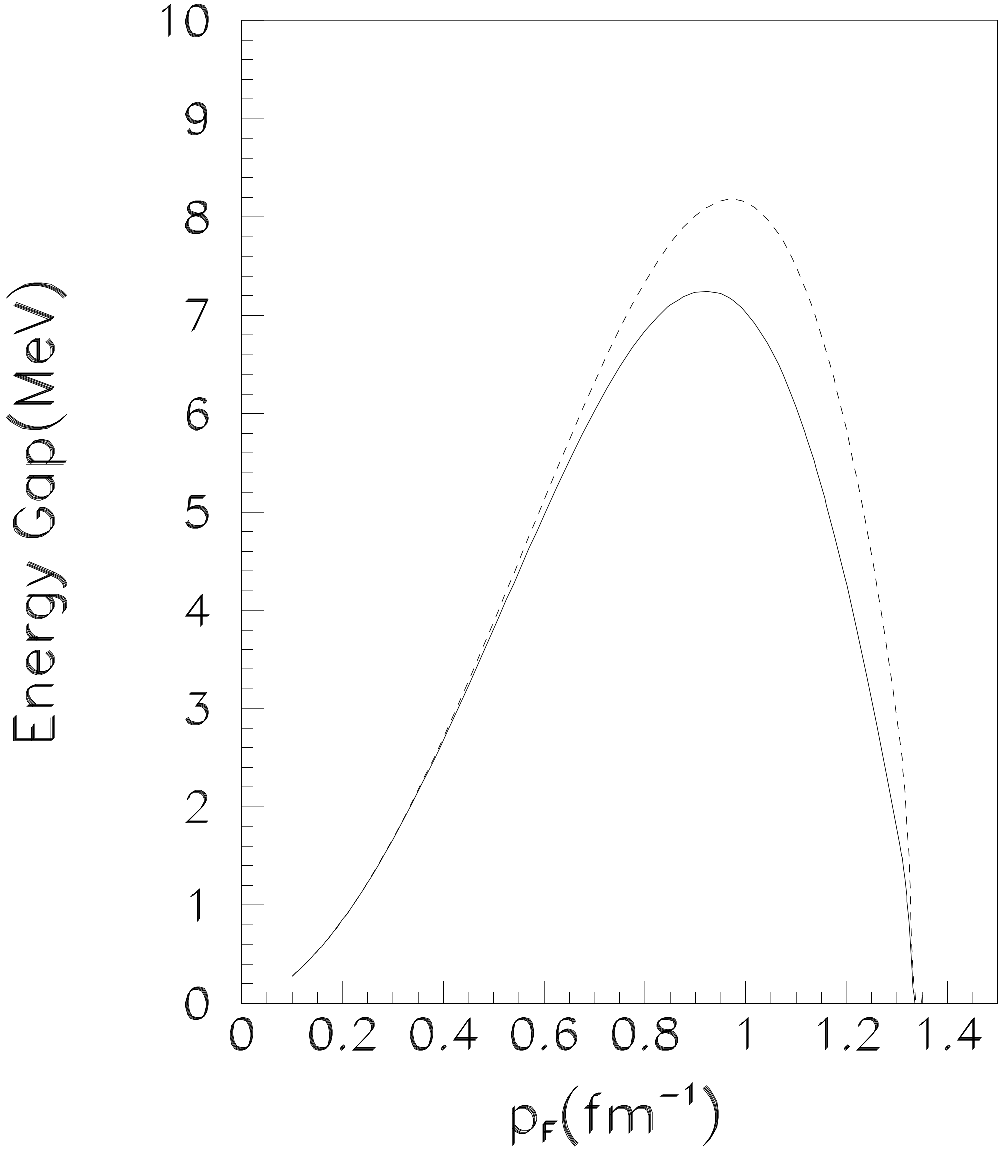
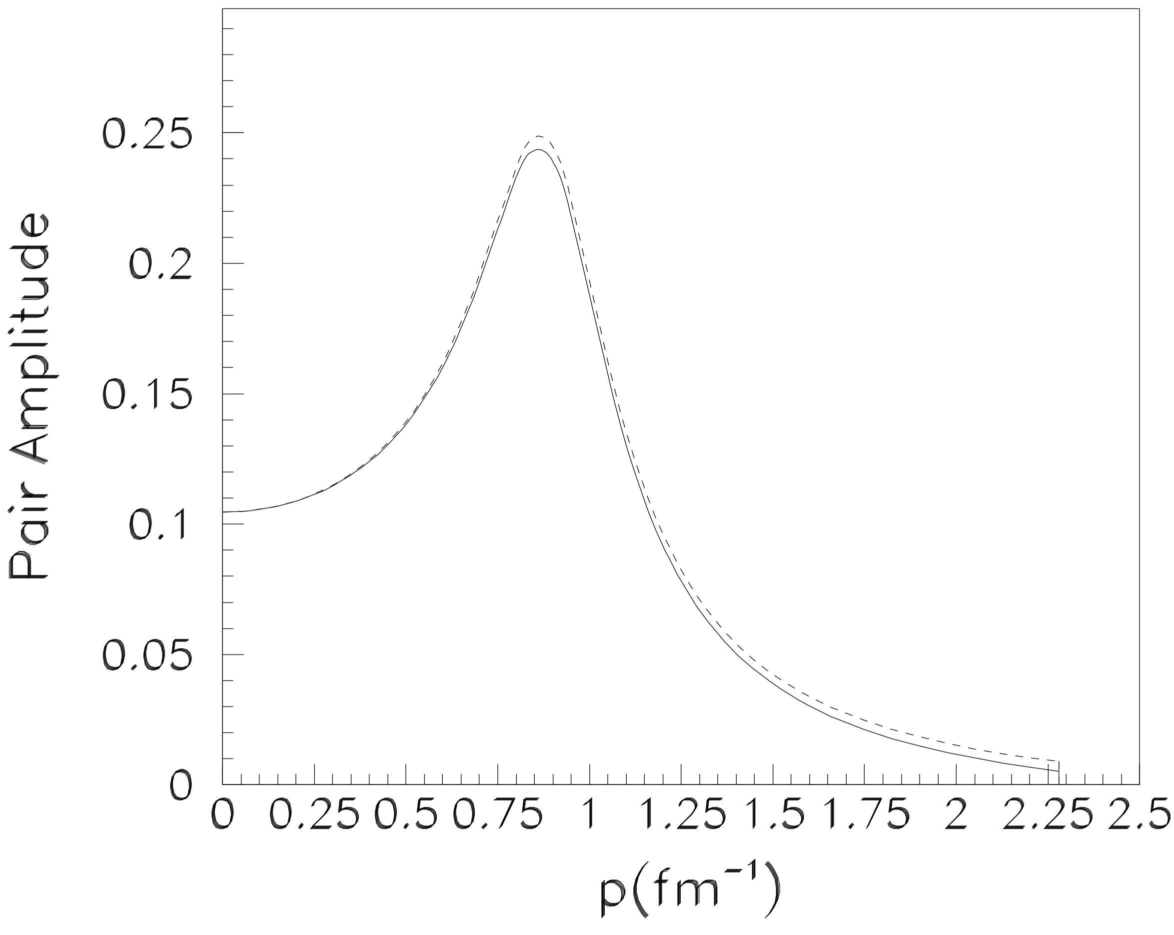
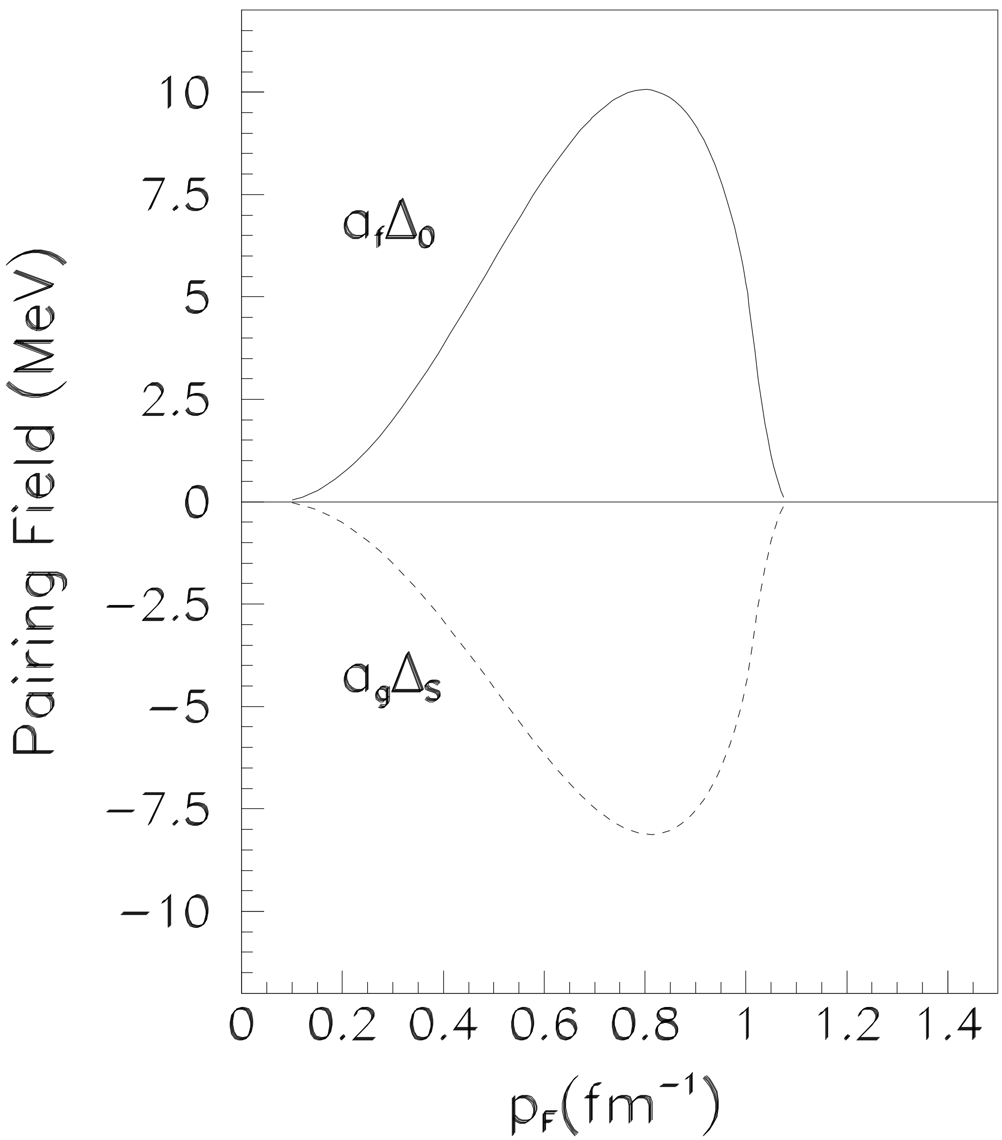


Fig.3



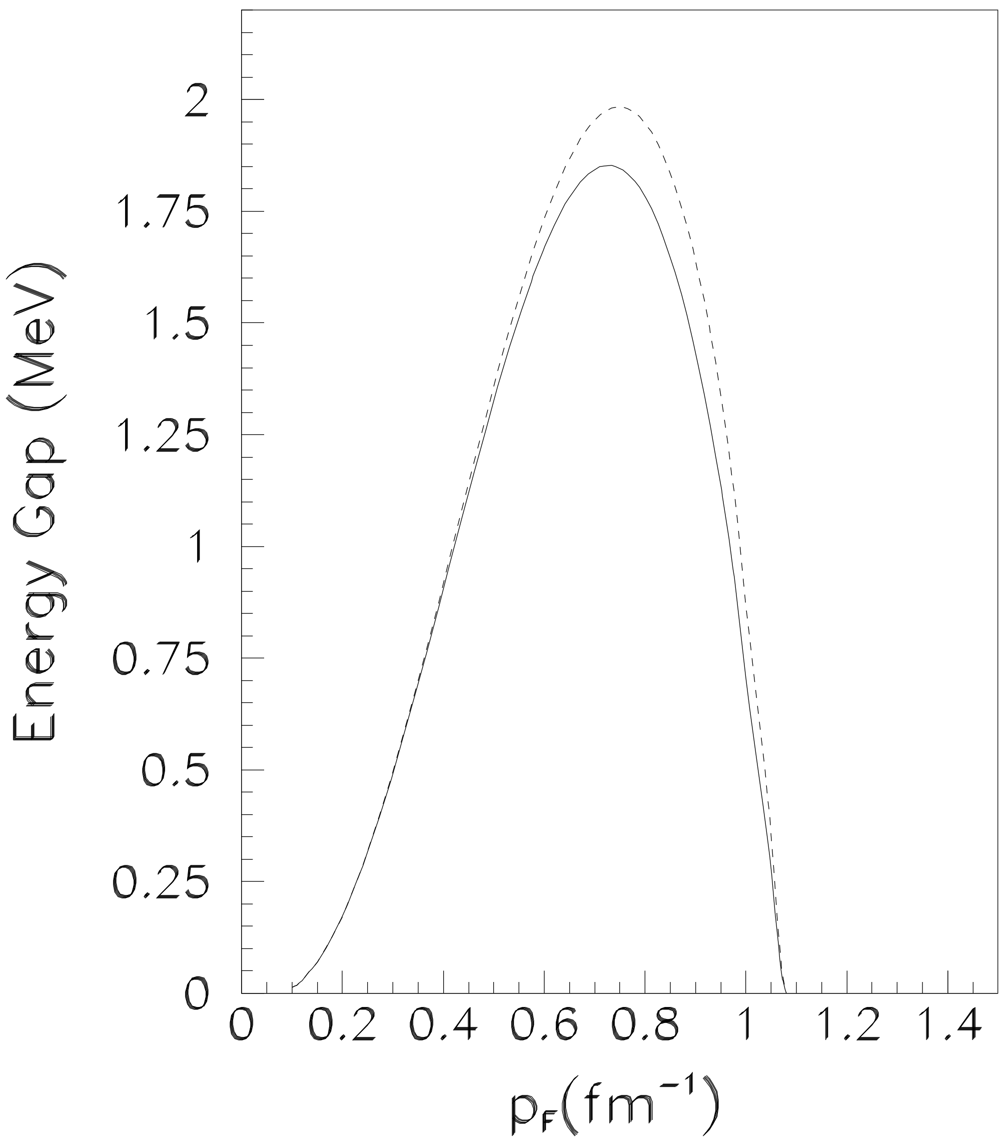
Pair Amplitude
0.25
0.2
0.15
0.1
0.05
0
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
p(fm⁻¹)

Fig.4



Pairing Field (MeV)
10
7.5
5
2.5
0
−2.5
−5
−7.5
−10
$a_f\Delta_0$
$a_g\Delta_s$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
$p_F(fm^{-1})$

Fig.5



Energy Gap (MeV)
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
$p_F(fm^{-1})$

Fig.6

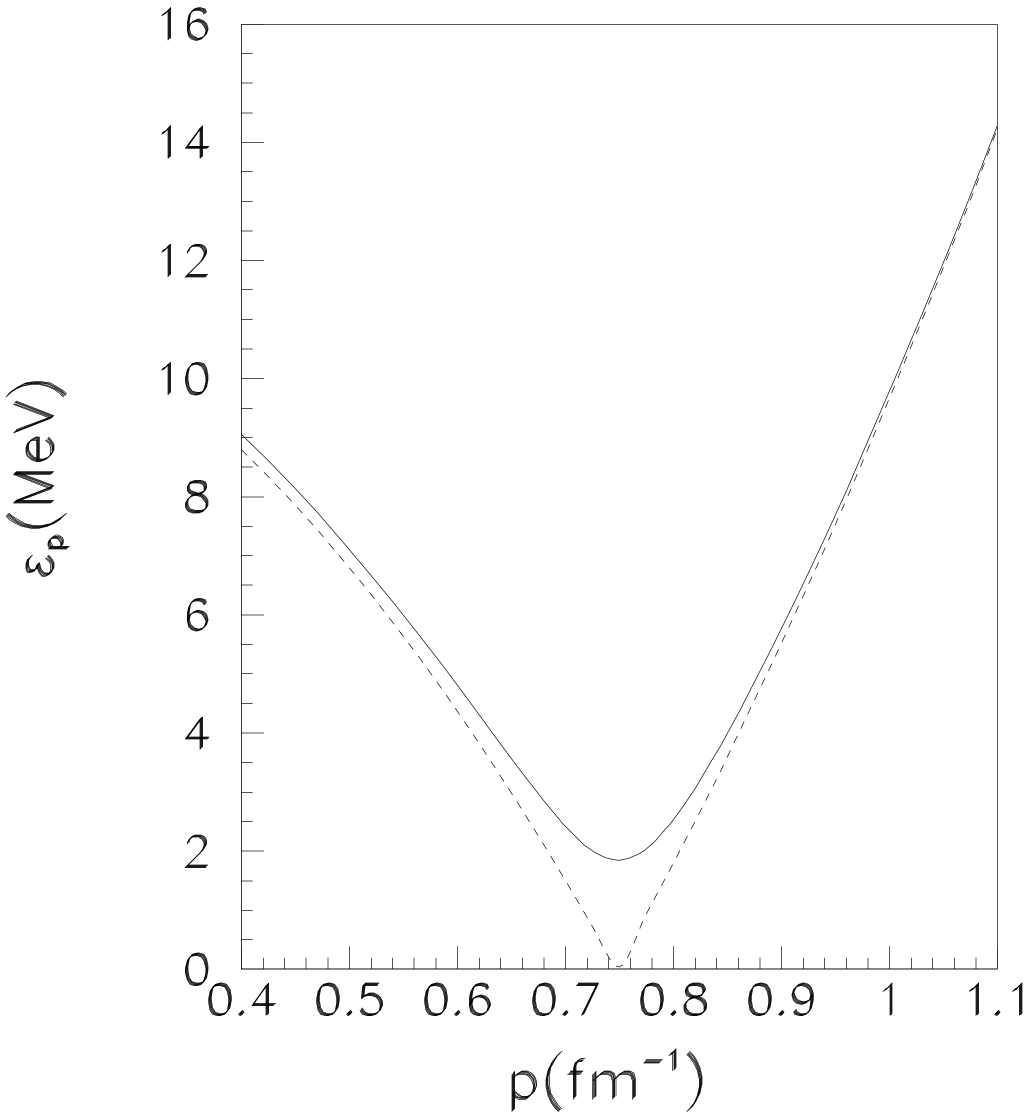


Fig.7

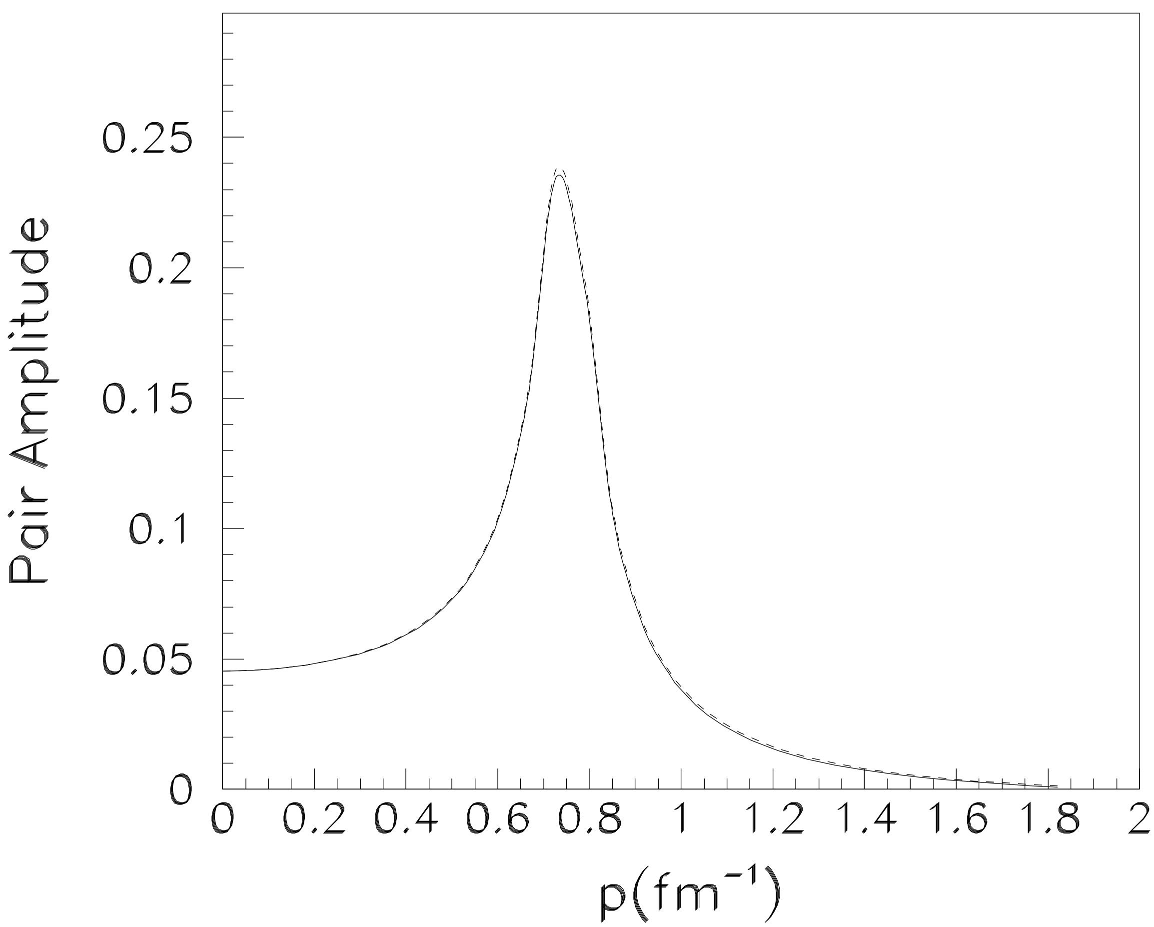

Pair Amplitude
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p(fm⁻¹)

Fig.8